Stable Homology-Based Centrality Measures for Weighted Graphs

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High-Dimensional Datasets

Representation of Numbers

- **Nodes** to embody a group of numbers.

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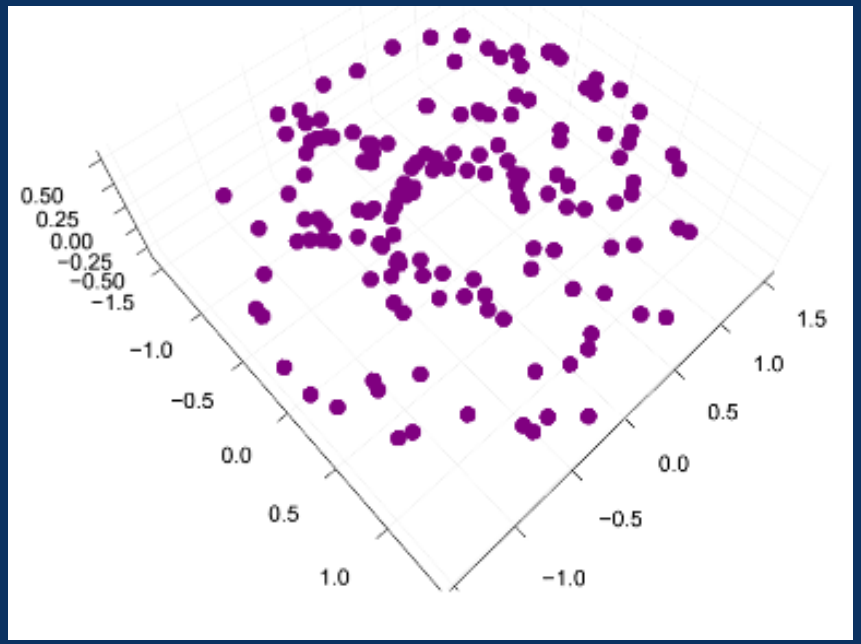
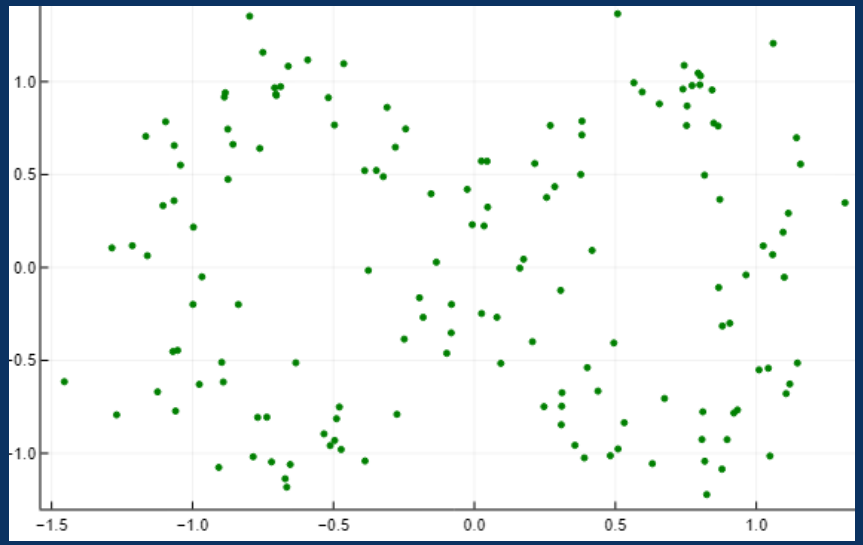
- Points in a metric space.





Topological Data Analysis

- Properties from the shape of an object.

Algebraic Topology $\Rightarrow$ Simplicial Homology

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- Clusters and Cycles

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Topological Data Analysis

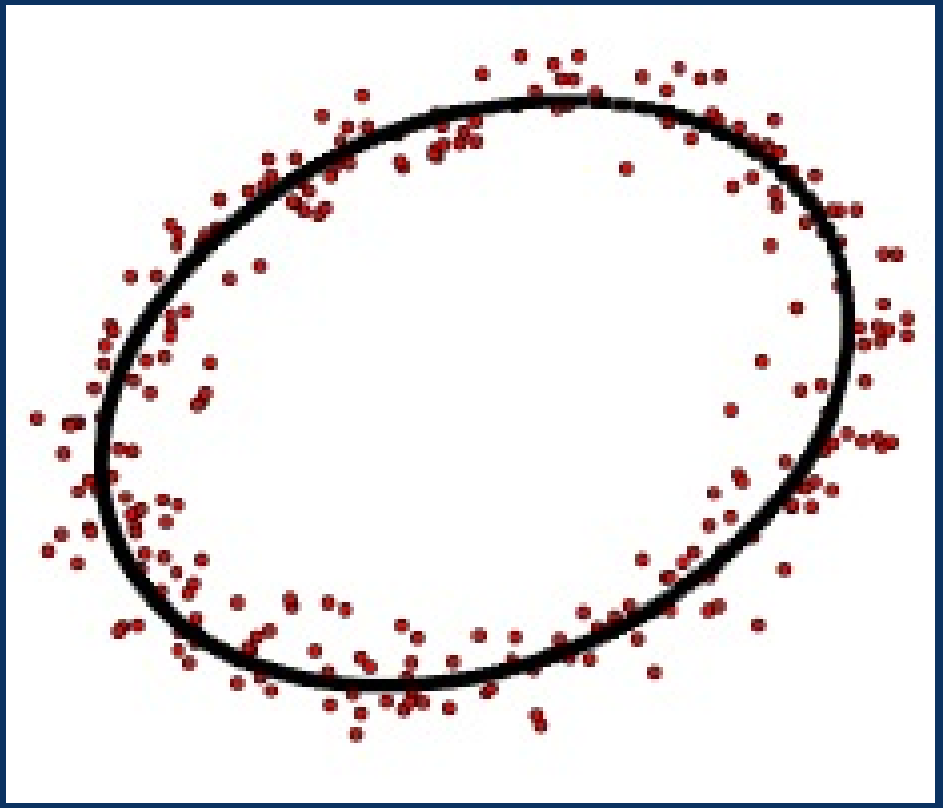
- Properties from the shape of an object.

- Clusters and Cycles

Examples of cycles are holes and voids.

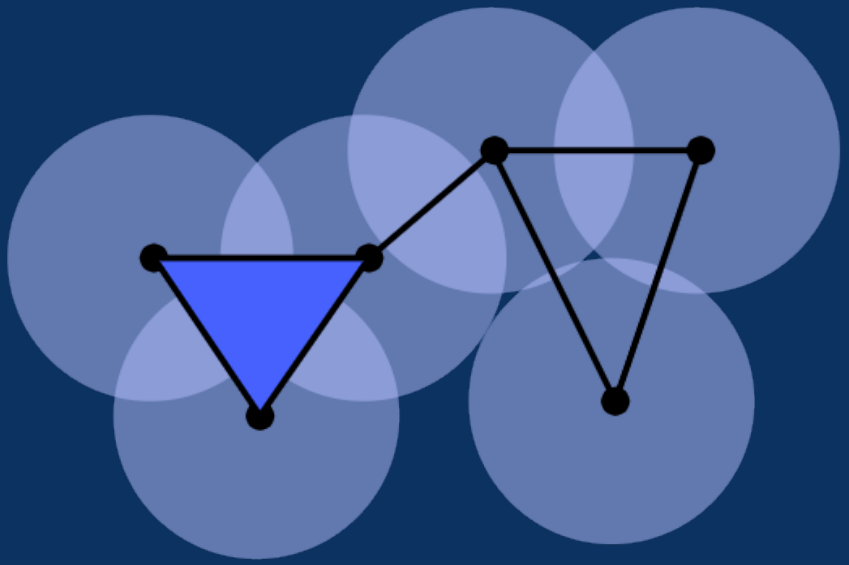
- Focus of our study.

Holes in Data



Relationship Between Two Points

- Connect any two points ϵ units apart.



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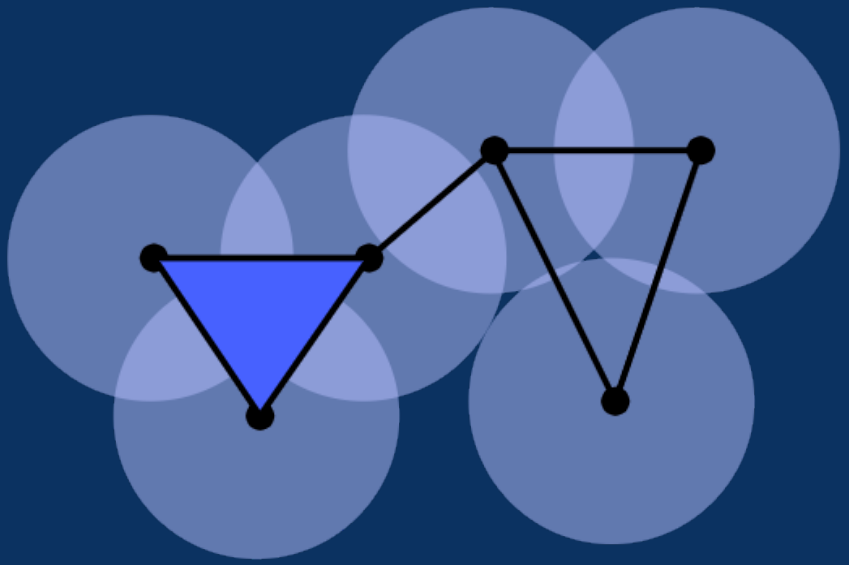
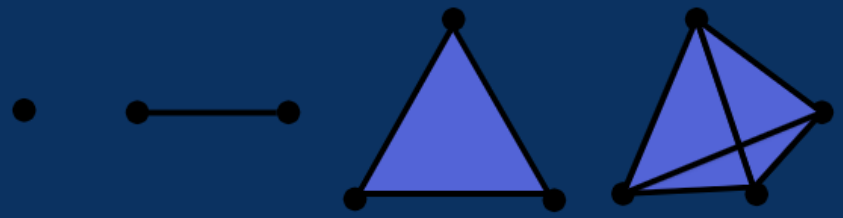


- The resulting structure is a **simplicial complex** $S(\epsilon)$.

Generalization of graphs

Features in Simplicial Complexes

- **Simplices**



Features in Simplicial Complexes

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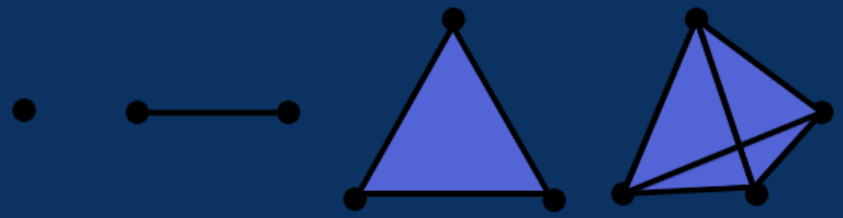
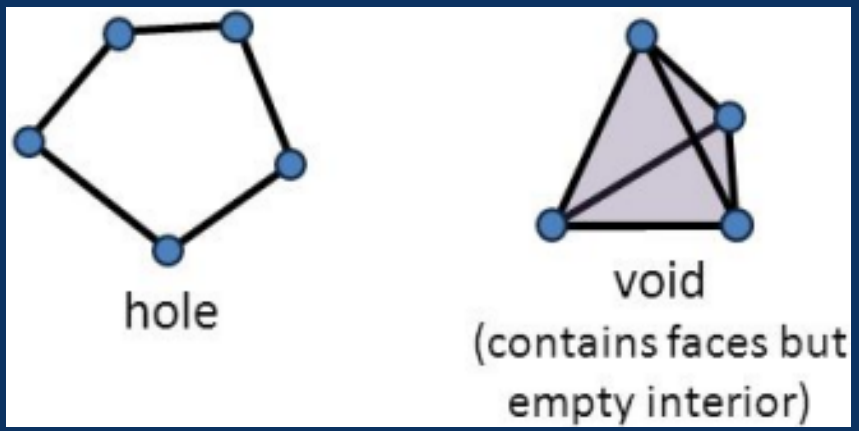
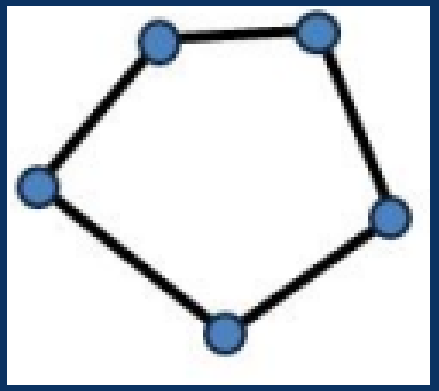


- Cycles



Life and Death Situation

- Living Cycles



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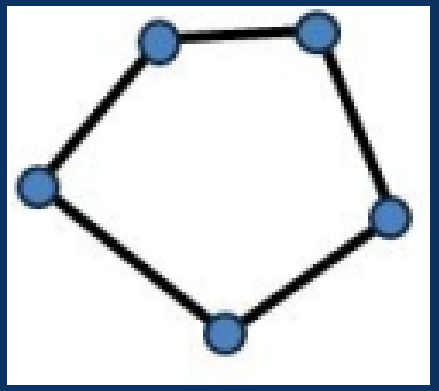
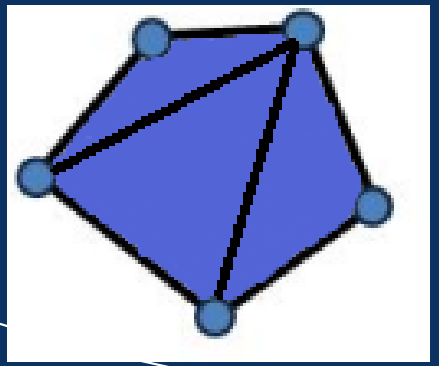
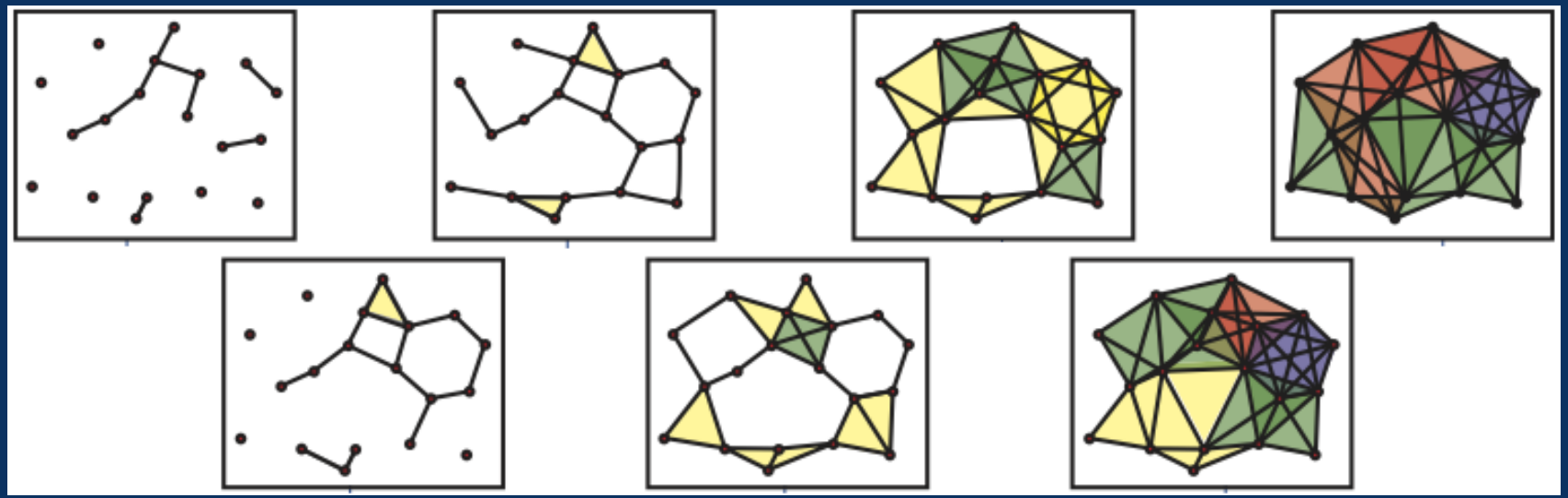


- Dead Cycles



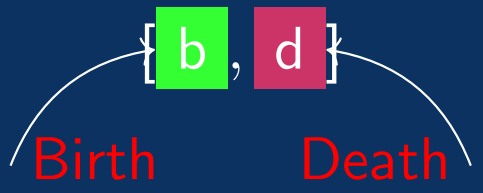
Define a criterion for simplices and increase ϵ.

Sequence of Simplicial Complexes



Persistence of Cycles

- Ordered Pairs



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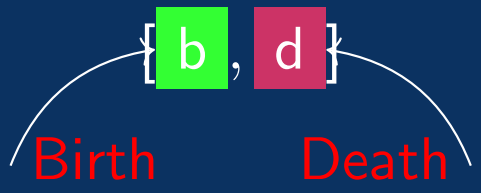
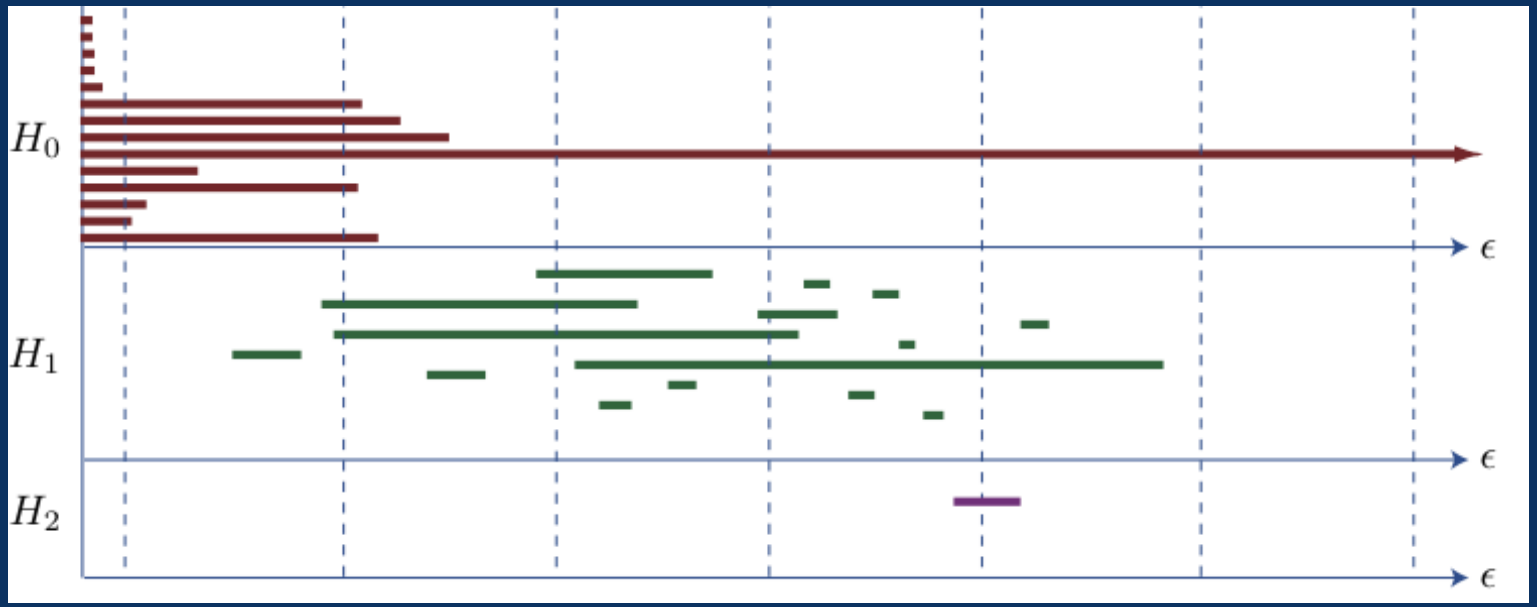
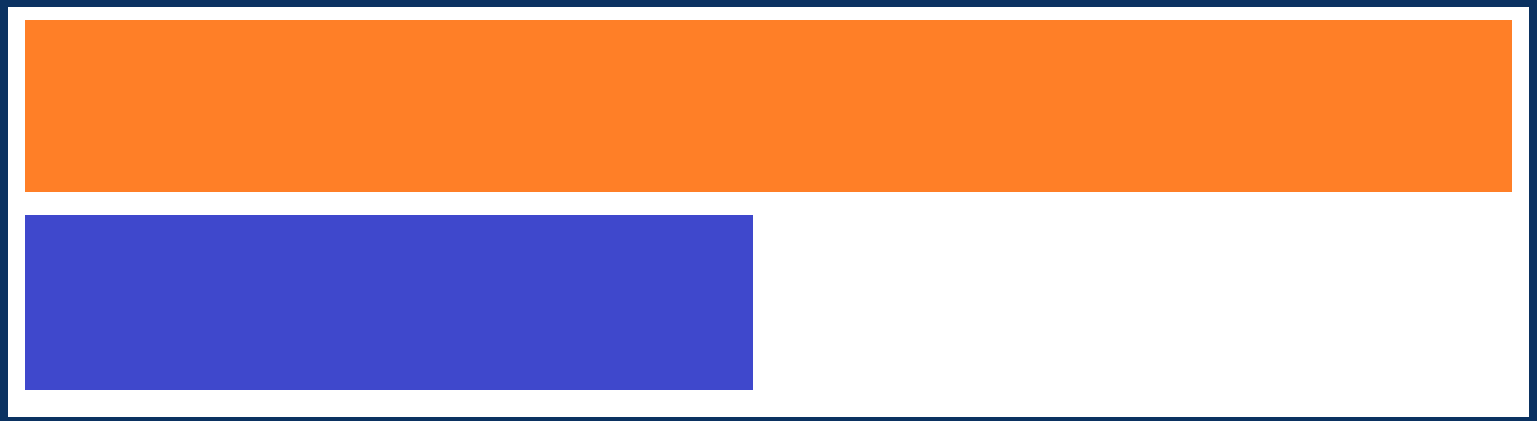


- Barcode



Quantifying Importance of a Cycle

- More persistent cycles are more important.



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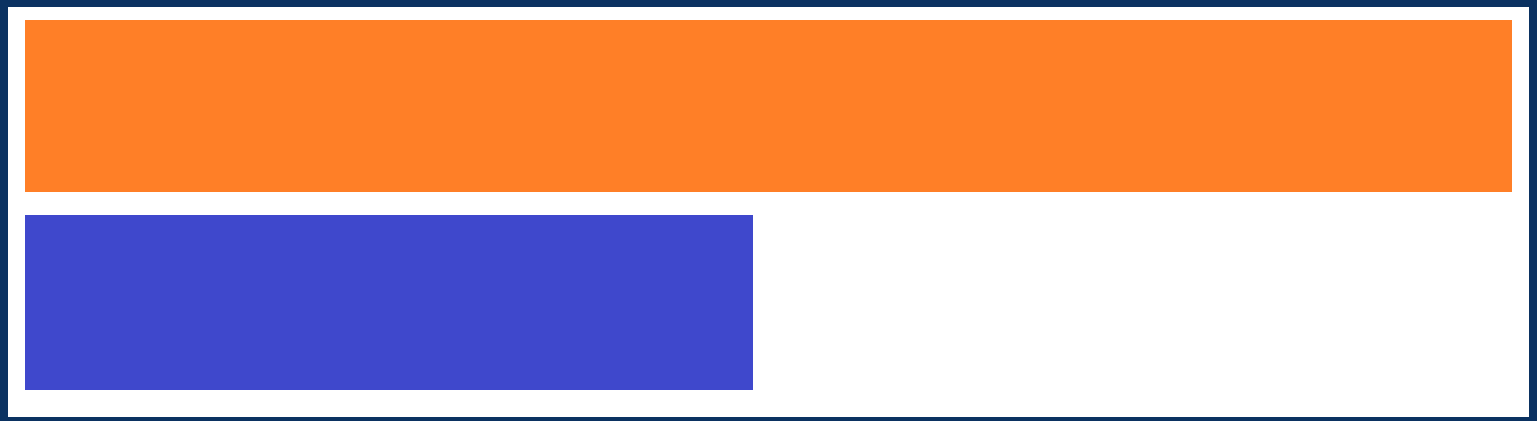


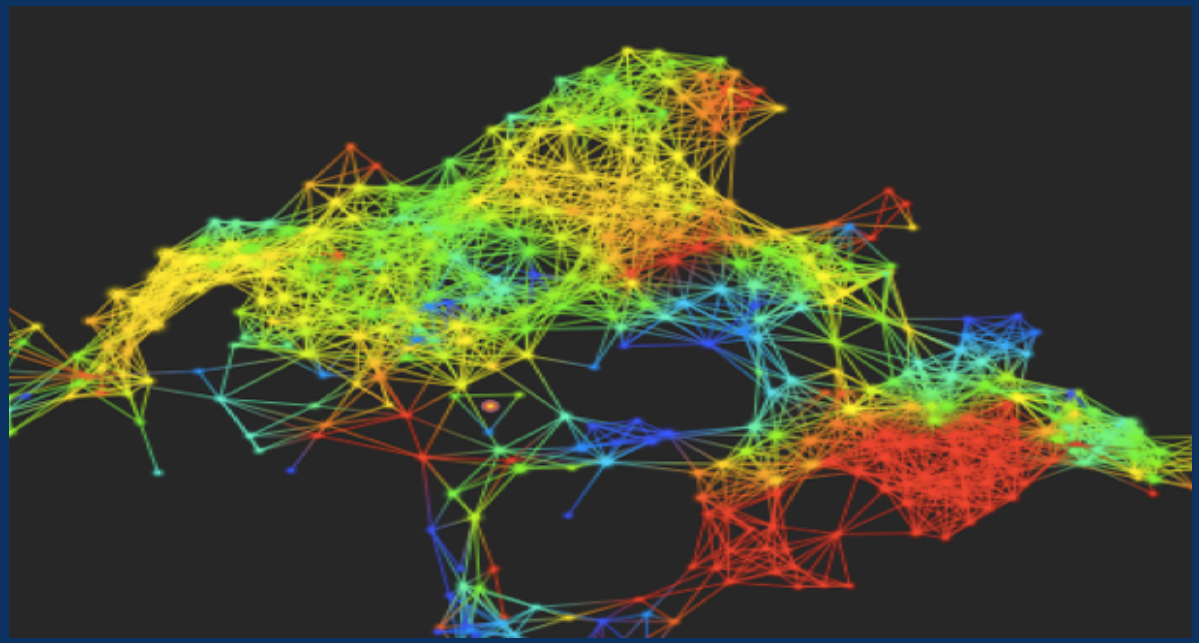
- Short-lived cycles are important in estimation problems.

Curvatures for surfaces

Centrality Measures in Graph Theory

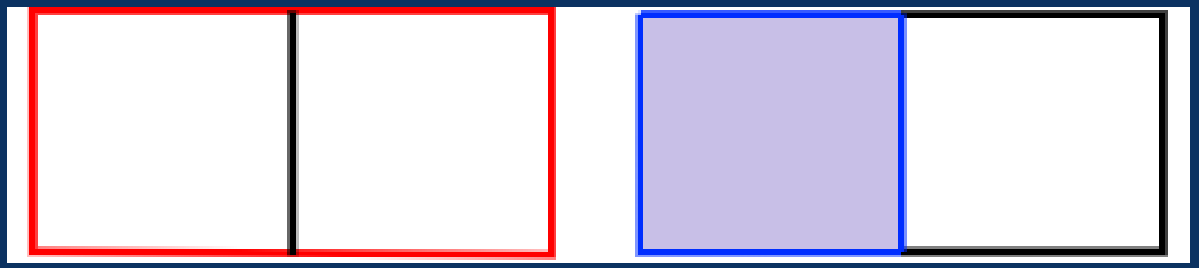
- Degree Centrality
- Betweenness Centrality
- Loop Centrality

Importance based on the number of walks passing the loop.



Merging Homological Cycles

- Two cycles merge if their sum is a sum of dead cycles of the same dimension.



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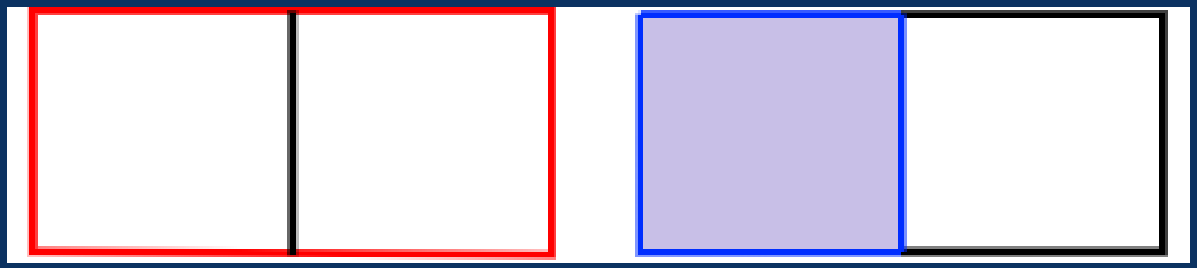

Cycle's Merging Information

Merging Class Merging Time

Proposed Centrality Measures

Centrality of a Cycle

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Centrality of a Cycle

- Persistence of Merging Cycles

$$a\left[\epsilon - b(\sigma)\right] + \sum_{\varsigma \in M[\sigma,\epsilon]} cP(\varsigma)$$

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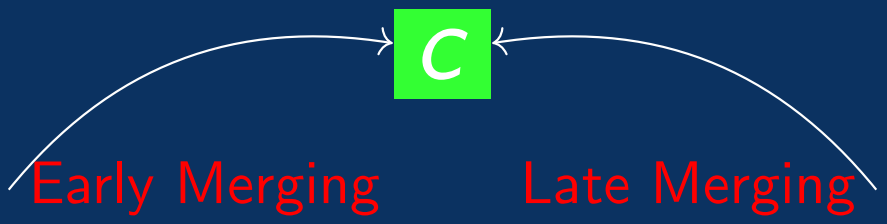
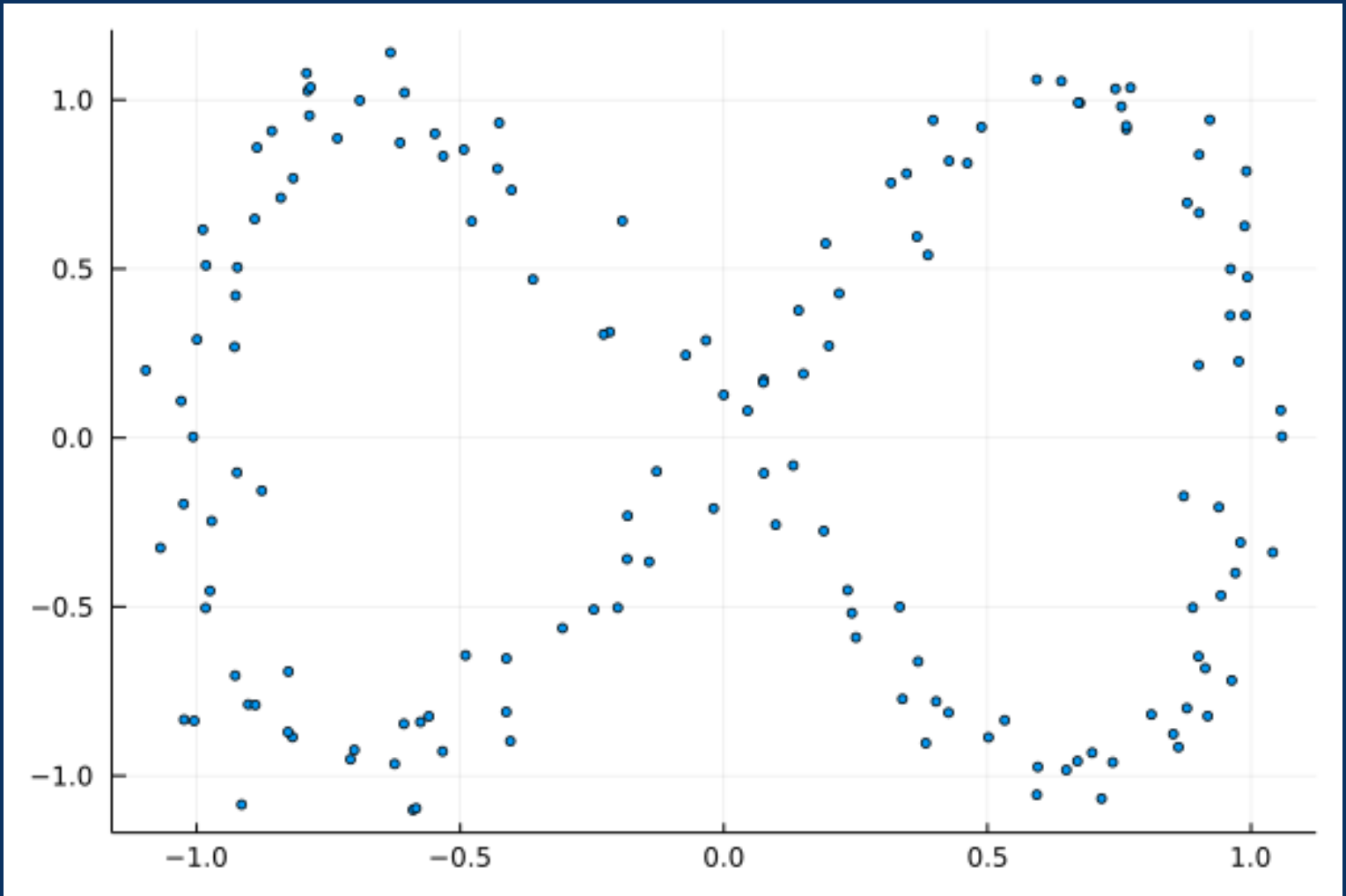
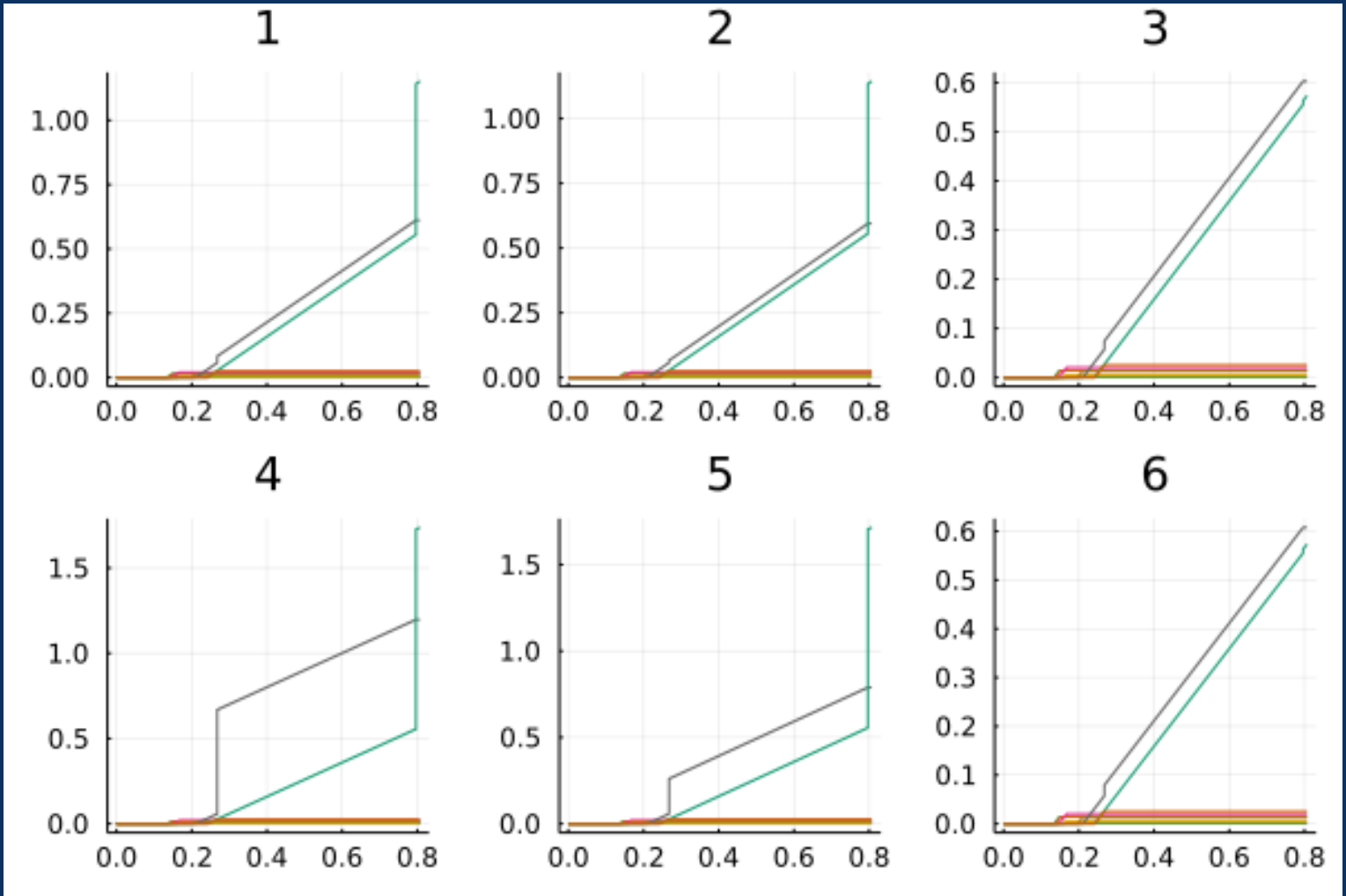


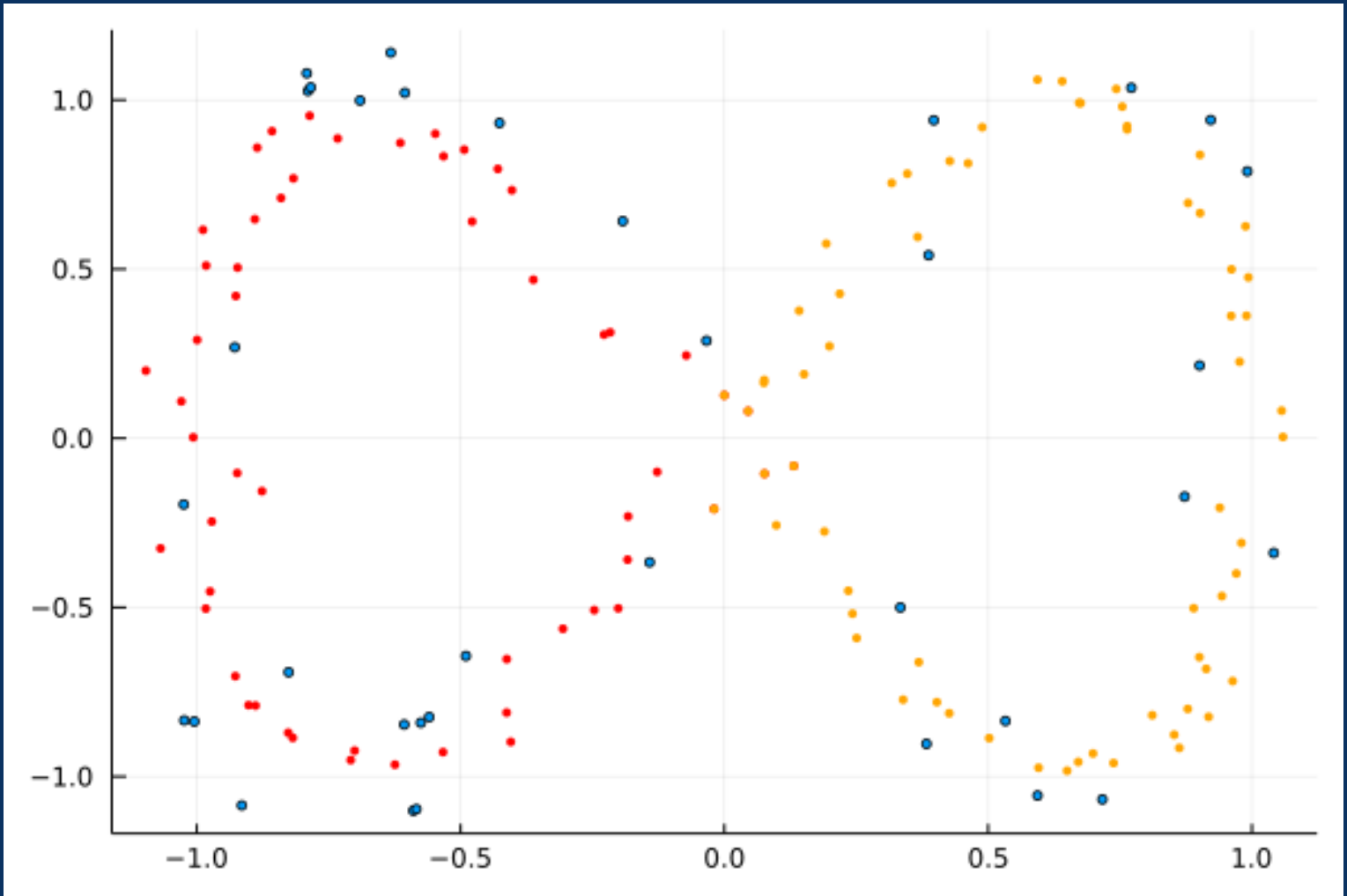
Application of Measures



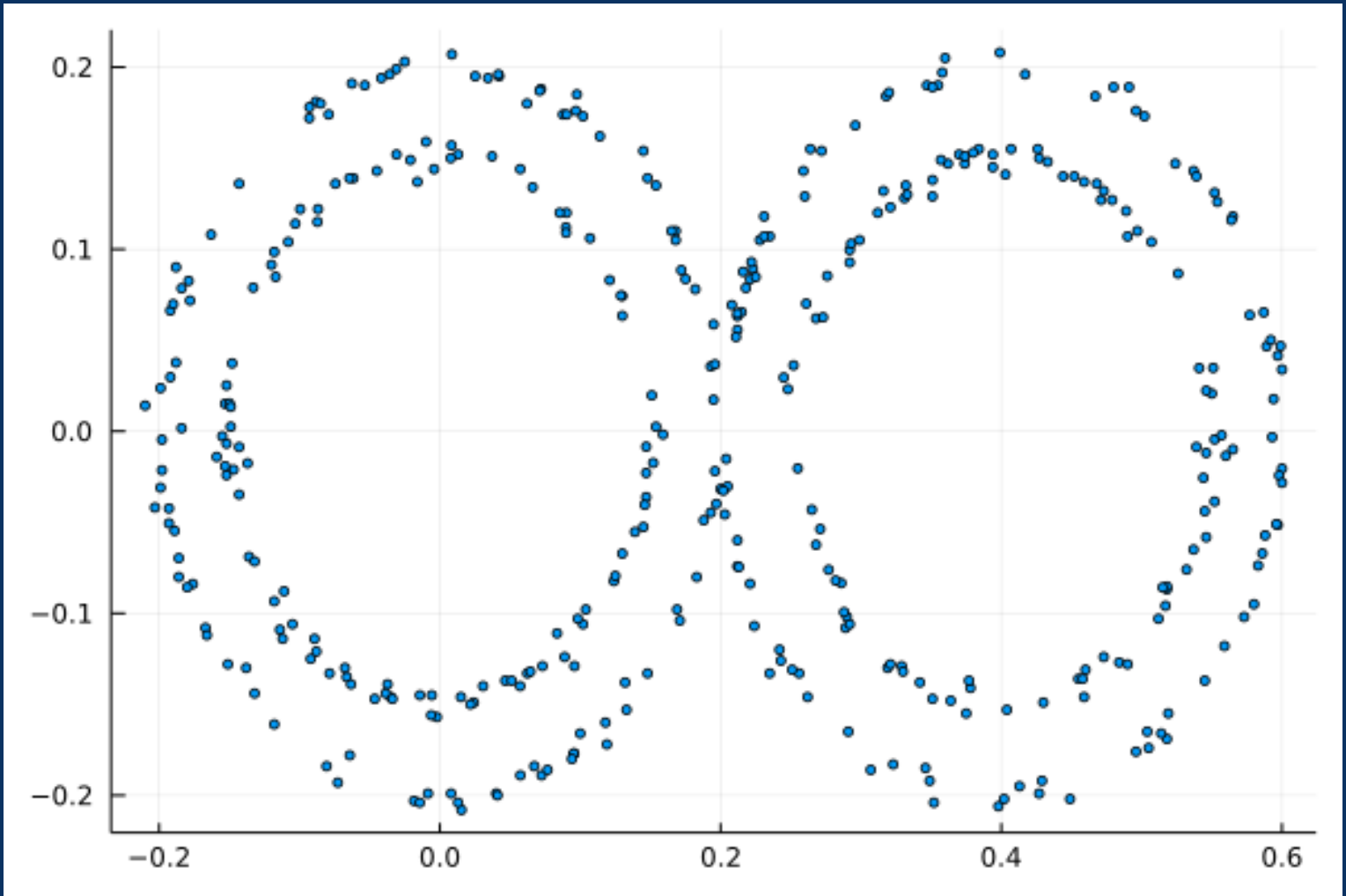
Representation of Measures



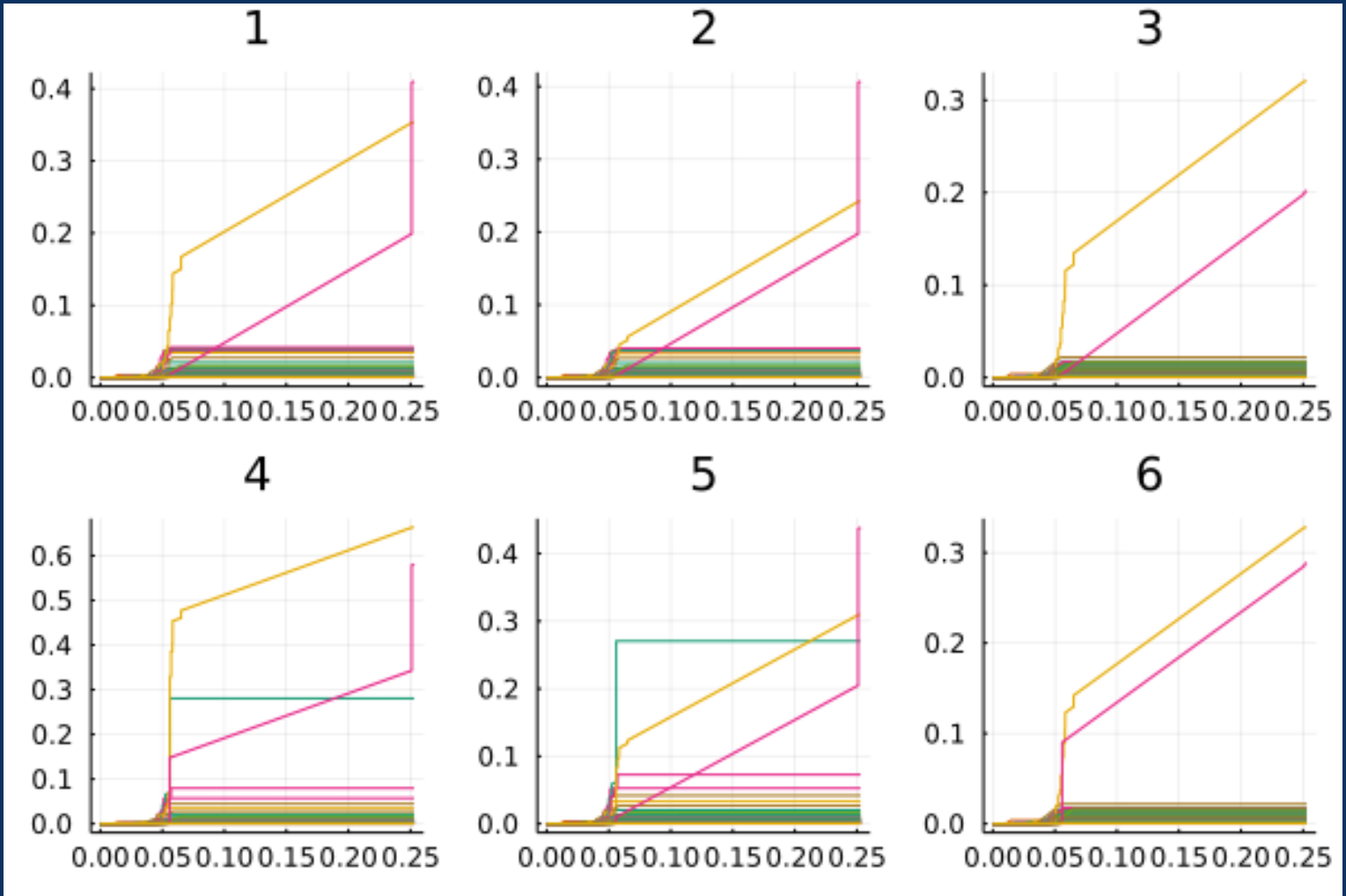
Expectation Meets Reality



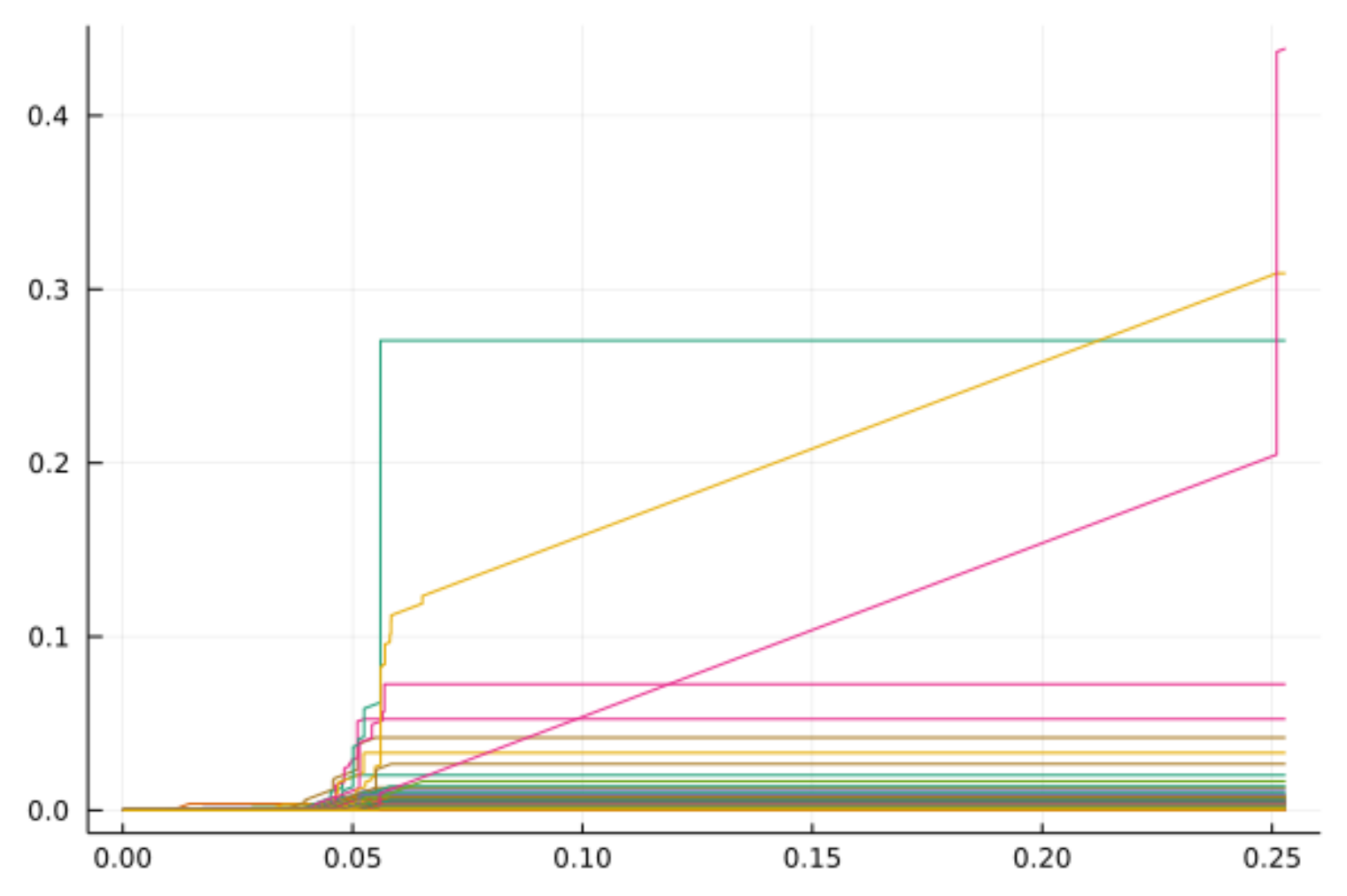
More Than Meets the Eye



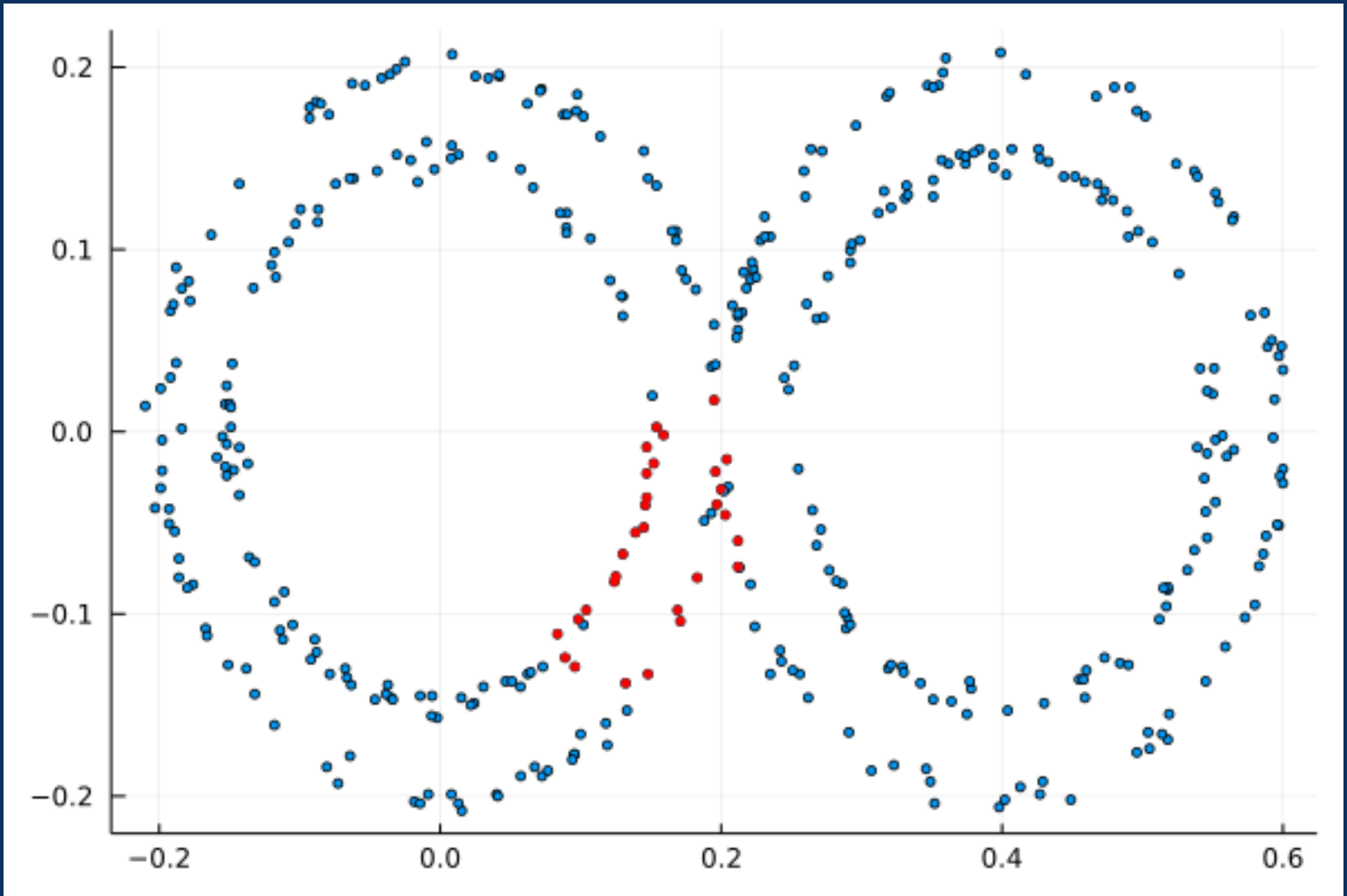
What does the Plot Say?



Better Late Than Never



Does Size Matter?



Conclusion

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  - Captures other properties

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  - Captures other properties
- Merging makes a difference.

Thank You!